Key

Worksheet Math 1316 -Plane Trigonometry

Unit 2: Section 3.4- Linear & Angular Speed

Formulas:

Arc Length Formulas:

1. $S = r \cdot \theta$

r = radius
θ = theta (angle)

2. $V = r \cdot \omega$

Liner Speed Formulas:

1. $V = \frac{S}{t}$

2. $V = \frac{r \cdot \theta}{t}$ ← arc length

Why?

b/c $S = r \cdot \theta$ ← arc length

Angular Speed Formulas:

1. $\omega = \frac{\theta}{t}$

2. $V = \frac{r \cdot \theta}{t}$

When solving problems always use radians unless specified

① what is given?

Converting Problems: ↑ 2π ↑ angular speed

The Earth completes **1 full rotation** in **24 hours**.
The radius of the Earth is **6400 km.**

1. Convert the angular speed to radians per hour. $\frac{rad}{hrs} \leftarrow \frac{rotation}{hrs}$

$$\frac{1\ rotation}{24\ hrs} \rightarrow \frac{2\pi}{24\ hrs} = \boxed{\frac{\pi}{12}\ rad/hr}$$

2. Find the linear speed (km/hr) at the Earth's surface. linear speed

$$V = rw \rightarrow V = 6400km\left(\frac{\pi}{12}\right) \qquad V = r \cdot w$$

$$V = \frac{1600\pi}{3} = \boxed{1675.5 km/hr} \qquad \uparrow \text{angular speed}$$

A space station completes **3 rotations in 6 hours.** ← angular speed
The radius of its circular path is **8000 km.**

1. Convert the angular speed to radians per hour.

$$\frac{3\ rotations}{6\ hours} \rightarrow \frac{3(2\pi)}{6} = \boxed{\pi\ radians/hr}$$

2. Find the linear speed in km/hr. $V = w \cdot r$

$$V = 8000km(\pi) = 8000\pi = \boxed{25{,}133 km/hr}$$

A planet completes **5 revolutions in 10 hours.** ← angular speed
The radius of the planet is **9,000 km.**

1. Find the angular speed in radians per hour.

$$\frac{5\ revolutions}{10\ hours} = \frac{5(2\pi)}{10} = \boxed{\pi\ rad/hr}$$

2. Find the linear speed at the surface. (km/hr) $V = r \cdot w$

$$V = 9000km(\pi) = 9000\pi = \boxed{28{,}274 km/hr}$$

① Draw diagram

Practice Problems:

arc length

$S = r \cdot \theta$ given:

1) A circle has a radius of 4 inches. Find the length of the arc intercepted by the central angle of 240°

$r = 4\text{in}$

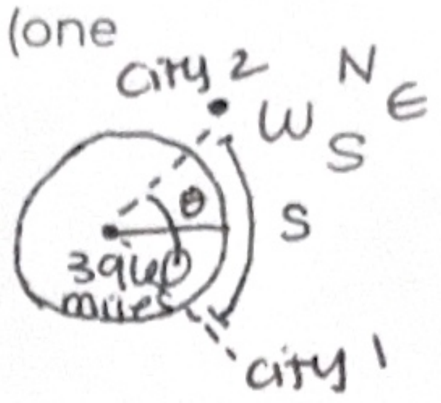
$\theta = 240°$

② convert degrees → radians

$240° = \frac{\pi}{180°} = \frac{240\pi}{180} = \frac{4\pi}{3}$ $S = 4\left(\frac{4\pi}{3}\right) = \boxed{\frac{16\pi}{3} \approx 16.76 \text{ inches}}$

both are correct

2) Find the distance between the cities. Assume that Earths sphere has radius of 3960 miles and the cities are on the same longitude (one city is due north of the other)



City 1: 26° S

City 2: 31° N

① Draw

② what is given
- radius = 3960

③ central angle?
- convert

$26° + 31° = 57°$

$\theta = 57° \rightarrow 57° \cdot \frac{\pi}{180°} = \frac{57\pi}{180} = \frac{19\pi}{60}$

④ distance

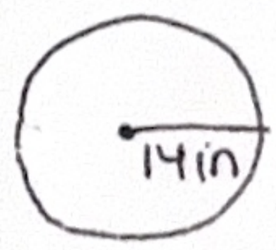
$S = 3960\left(\frac{19\pi}{60}\right)$

$= \boxed{1254\pi \approx 3939 \text{ miles}}$

3) A wheel of a radius is 14 in. and is rotating at a speed of 0.5 radians/sec

what do we have already → angular speed (w)

Draw:



Find: the angular speed in revolution per minute (RPM)

$0.5 \frac{\text{radian}}{\text{sec}} \rightarrow \frac{\text{revolution}}{\text{per min}}$

① convert rad to rev.

$\frac{0.5}{2\pi} = \frac{1}{4\pi} \frac{\text{rev}}{\text{sec}}$

② seconds → minutes

$\frac{1}{4\pi} \cdot 60 \rightarrow \boxed{\frac{15}{\pi} \approx 4.77 \text{ RPM}}$

60 second in a minute

Find: the linear speed in inches/seconds

$V = r \cdot w$

$V = 14(0.5) = \boxed{7 \text{in/sec}}$

$\rightarrow 14\text{in} \cdot 0.5 \frac{\text{radian}}{\text{sec.}}$